Direction Of Arrival (DOA) Estimation For Radars In Near-Field Regions

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Abstract— **The Direction of Arrival (DOA) estimation in the near-field has received significant attention in recent years[1] due to its wide range of applications in various fields, including gesture recognition[2], radar systems, and acoustic imaging. Multiple DOA algorithms have been developed for far-field regions, such as Beamforming[3] and multiple signal classification (MUSIC)[4]. These algorithms depend on far-field assumptions, such as simplifying electromagnetic waves as parallel planes. Conversely, in the near-field regions, the assumption of plane waves is not applicable; electromagnetic waves exhibit a spherical pattern. As a result, traditional DOA estimation algorithms based on far-field assumptions fail. To bridge this gap, we introduce a modification to these algorithms in order to turn them viable in the near-field regions. Instead of proposing a new DOA algorithm for the near-field region, the goal of this paper is to propose a modification to the covariance matrix used in various far-field DOA algorithms to make them applicable in the near-field regions. We first introduce the basic concepts of DOA estimation[3] and a couple of far-field DOA estimation algorithms. Then, we discuss the challenges facing these algorithms in the near-field regions. We proceed with presenting a modification to these algorithms. We finish by demonstrating the improvement through empirical evaluations and comparisons.**

Keywords— ***DOA, near field, gesture recognition, radar systems, DOA estimation algorithms, far-field assumptions, MUSIC, Beamforming, covariance matrix.***

I. Introduction

The estimation of the Direction of Arrival (DOA) of signals is an important problem in array signal processing, with numerous applications in fields such as gesture recognition, radar systems, and acoustic imaging. In recent years, there has been a growing interest in the estimation of DOA in the near field, where the distance between the source and the sensor is comparable to the wavelength of the signal. This region presents unique challenges for DOA estimation algorithms based on far-field assumptions, such as the assumption of planar wavefronts, which may lead to inaccurate DOA estimates.

To address this challenge, new techniques are required to accurately estimate DOA in the near field. In this paper, we begin by presenting classical DOA estimation algorithms operating optimally in far-field regions. Subsequently, we showcase their performance degradation in near-field regions. To bridge this gap, we introduce our modified approach for these algorithms, demonstrating their enhanced performance in near-field conditions.

The MUSIC algorithm is a popular subspace-based method that estimates the DOA by exploiting the eigenstructure of the signal subspace. However, in the near field, the performance of the MUSIC algorithm degrades due to the violation of far-field assumptions.

Beamforming is another spatial filtering technique that can be used for DOA estimation in far-field regions. In this technique, the received signals are combined with appropriate weights to form a beam that maximizes the signal-to-noise ratio (SNR) of the signal of interest.

In summary, this paper provides a comprehensive overview of the state-of-the-art techniques for DOA estimation in far-field regions and showcases modified versions of the MUSIC and beamforming algorithms. The findings of this paper can provide a useful approach to turn known far-field DOA estimation algorithms (especially super-resolution algorithms) applicable in the near-field regions.

II. Model Description

The DOA estimation problem involves M sources that emit electromagnetic signals toward an antenna array consisting of N elements.

A. Assumptions

1) Isotropic and linear transmission medium: There are M sources that emit M signals which travel through the medium and impinge onto an N-element antenna array. The isotropic and linear nature of the medium ensures that the physical properties of the medium remain the same in all directions, and that signals can be linearly superposed at any point. This property of the medium ensures that the propagation properties of the signals do not change with the Direction of Arrival (DOA), and that signals received by any element of the M-element array can be computed as a linear superposition of the M signal wavefronts generated by the M sources. Furthermore, the gain of each antenna or sensor element is assumed to be one.

2) Narrowband assumption: All M signals emitted by the M sources share the same carrier frequency, with frequency contents concentrated around the carrier frequency f_c. Mathematically, an instantaneous signal from any of the sources can be expressed as:

$$s_i^r(t) = \alpha_i(t)\cos\left[2\pi f_c t + \beta_i(t)\right] \; ; \; 1 \le i \le M \qquad (1)$$

3) AWGN channel: The noise is assumed to be of a complex white Gaussian process, with zero mean and variance σ_N^2 . In addition, the noise signals are uncorrelated with either the source signals or the noise in other antennas.

B. Uniform Linear Array (ULA)

A uniform linear array (ULA) consists of N identical antennas that are placed on a straight line that is separated from each other by an equal distance denoted by d. Let the distance between the i^{th} source and the first element be

M_i, and the angles indicating the position of the source relative to the central antenna be θ_i. In addition, let the signal that the n^{th} antenna receives by the i^{th} source be $s^r_{ni}(t)$, and the complex signal by $s_{ni}(t)$.

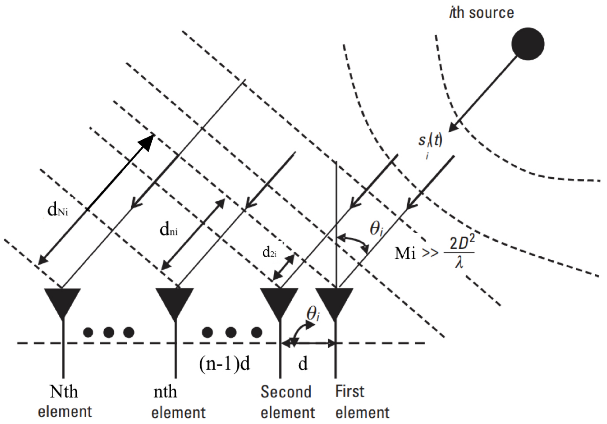

Fig. 1. DOA estimation of M sources with ULA of N elements.

The signal received by the nth antenna, denoted by $x_n(t)$, can be expressed as the sum of the source signals received by antenna *i*, assuming linear transmission. In addition, we will account for noise in the signal, which will be denoted as $n_n(t)$:

$$x_n(t) = \sum_{i=1}^{M} s_{ni}(t) + n_n(t)$$

$$= \left[e^{-j(n-1)\mu_i} \cdots e^{-j(n-1)\mu_d} \right] \begin{bmatrix} s_{11}(t) \\ \vdots \\ s_{M1(t)} \end{bmatrix} + n_n(t) \quad (2)$$

Where:

$$\mu_i = \frac{2\pi f_c d \sin(\theta_i)}{c} = \frac{2\pi d \sin(\theta_i)}{\lambda} \quad (3)$$

In a matrix form, (2) can be written as:

$$x(t) = \left[a(\mu_1), \ldots, a(\mu_M) \right] \begin{bmatrix} s_{11}(t) \\ \vdots \\ s_{M1}(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ \vdots \\ n_N(t) \end{bmatrix} \quad (4)$$

$$x(t) = As(t) + n(t)$$

III. Classic Beamforming

The fundamental concept of beamforming techniques is to direct the array in a specific direction and measure the output power. By aligning the directed direction with the Direction of Arrival (DOA) of a signal, the maximum output power is obtained. Thus, Beamforming sequentially scans the various directions and determines the DOA as the angle at which the peak power is observed.

Given the knowledge of array steering vector, an array can be electronically steered in a similar manner to the mechanical steering of a fixed antenna. In addition to changing orientation, the shape of the array pattern can also be altered. By designing a weight vector w, the data received by the array elements can be linearly combined to generate a single output signal y(t):

$$y(t) = w^T x(t) \quad (5)$$

Given K samples of the antennas' readings $\{x(t_k)\}_{k=1}^{K}$, try to maximize the average output power which is:

$$P(w) = \frac{1}{K}\sum_{k=1}^{K} \left| y(t_k) \right|^2 = \frac{1}{K}\sum_{k=1}^{K} \left| w^T x(t_k) \right|^2$$
$$= \frac{1}{K}\sum_{k=1}^{K} w^T x(t_k) x^T(t_k) w = w^T \widehat{R_{xx}} w \quad (6)$$

Where R_{xx} is the revaluation of the covariance matrix.

The vector *w* is designed to steer the sample towards achieving maximum output power at the angle of arrival. By defining *w* as the negative delay each antenna receives relative to the first antenna, constructive interference is achieved when the delay corresponds to the actual delay. At this point, the sum, or power, of the antenna samples is maximized.

Scans angles $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and for each scanned angle calculate:

$$w = a(\mu(\theta)) = \begin{bmatrix} e^{j(1-1)\mu(\theta)} \\ \vdots \\ e^{j(N-1)\mu(\theta)} \end{bmatrix} \quad (7)$$

Whereas:

$$\mu(\theta) = \frac{2\pi d \sin(\theta)}{\lambda} \quad (8)$$

Then calculates $P(w)$, and returns the angle with the maximum value.

A. *Performance*

The performance of the Beamforming algorithm is assessed in the context of a single source operating at a frequency of 30 GHz (1 cm wavelength). The source is positioned at an angle of *θ=30°* relative to an antenna array consisting of five elements, each with a separation distance of 0.5 cm (half a wavelength). The antenna array exhibits a signal-to-noise ratio (SNR) of 20[dB]. The near-field to far-field transition distance is established as 8 cm based on the given parameters.

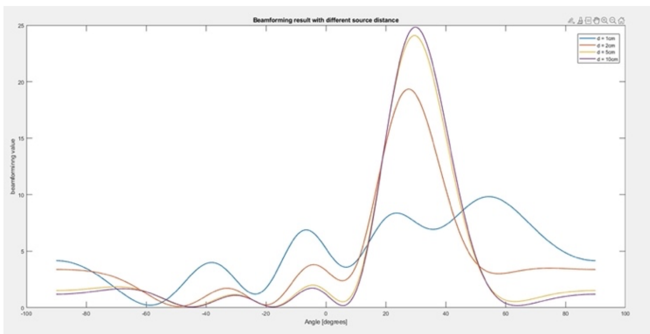

Fig. 2. Beamforming results at different source distances.

The results demonstrate that, at a distance of 10 cm, which exceeds the long-range threshold, the error rate is 0.1 degrees with a peak value in close proximity to 25. As the distance is reduced to 5 cm and the threshold is breached, the error rate increases to 0.5 degrees, while the peak value remains close to 25. When the distance is further reduced to 2 cm, a significant portion of the data points exceeds zero, and the peak value drops to less than 20. Additionally, the error rate increases to 2.5 degrees. When the distance is further reduced to 1 cm, the graph exhibits significant noise, with no discernible peak as previously observed. Instead, a new peak emerges with a value of 10, which is 2 units lower than those observed at distances above the threshold. The error rate at this distance is 24.83 degrees, and the algorithm can be considered faulty.

IV. MUSIC ALGORITHM

The Multiple Signal Classification Algorithm (MUSIC) is a high-resolution technique for estimating the direction of arrival of signals using eigenstructure analysis. The method involves manipulating the correlation matrix of the antenna samples and decomposing it into eigenvalues and eigenvectors. As the correlation matrix contains both signal and noise components, the resulting eigenvectors correspond to the steering vectors of the sources (with high eigenvalues) and the noise subspace (with low eigenvalues). The signal subspace and the noise subspace are orthogonal to each other, as they are both spanned by eigenvectors of the same matrix. In particular, the steering vectors of the sources are orthogonal to the noise subspace, with their product being zero.

Given M sources, N elements, and K samples with value X, the revaluation of the covariance matrix is:

$$\widehat{R_{xx}} = \frac{1}{K} XX^H \tag{9}$$

Sorting the N eigenvalues from the smallest to the largest, so the noise subspace E_N is composed of the N-M eigenvectors associated with the noise:

$$E_N = \left[v_M, v_{M+1}, \ldots, v_N \right] \tag{10}$$

Then scans angles $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, and for each scanned angle the steering vector corresponding to this angle is defined as:

$$e(\theta) = a(\mu(\theta)) = \begin{bmatrix} e^{j(1-1)\mu(\theta)} \\ \vdots \\ e^{j(N-1)\mu(\theta)} \end{bmatrix} \tag{11}$$

Whereas:

$$\mu(\theta) = \frac{2\pi d \sin(\theta)}{\lambda} \tag{12}$$

Subsequently, we compute the product of the steering vector with E_N:

$$\mathrm{MusicSpectrum}(\theta) = \frac{1}{\left| e(\theta)^H E_N E_N^H e(\theta) \right|} \tag{13}$$

Lastly, the algorithm returns the angles associated with the highest values.

A. Performance

The performance of the MUSIC algorithm is assessed in the context of a single source operating at a frequency of 30 GHz (1 cm wavelength). The source is positioned at an angle of θ=30° relative to an antenna array consisting of five elements, each with a separation distance of 0.5 cm (half a wavelength). The antenna array exhibits a signal-to-noise ratio (SNR) of 20[dB]. The near-field to far-field transition distance is established as 8 cm based on the given parameters.

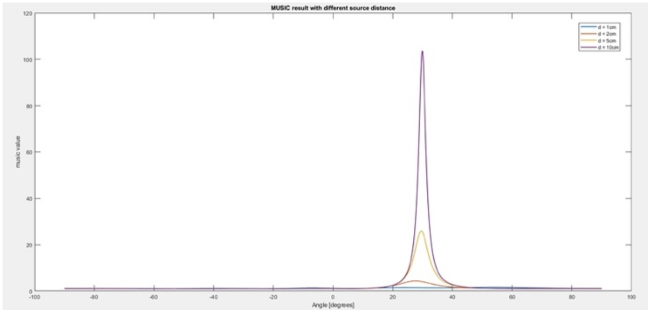

Fig. 3. MUSIC results at different source distances.

The results demonstrate that the error is 0.1 degrees and the peak value is approximately 100 at a distance of 10 cm, which is beyond the long-range threshold. As we move closer and pass the threshold at 5 cm, the error increases to 0.5 degrees and the peak is around 24. At 2 cm, it becomes apparent that most of the data points are greater than zero, and the peak value reduces to 4.3, while the error increases to 2.4 degrees. As we approach 1 cm, the graph becomes noisy with no distinctive peak, and a new peak emerges with a value of 1.6, which is significantly lower than the values observed at distances above the threshold. The error also increases to 24.4 degrees, indicating that the algorithm is no longer reliable. Additionally, it is worth noting that the beam's width widens as we get closer, which adversely affects the algorithm's resolution.

V. MODIFYING THE ALGORITHMS

The known algorithms are designed for sources in the far-field regions. This is evident in the computation of the algorithm, which relies on a steering vector that is solely dependent on the angle. In other words, each angle corresponds to a unique steering vector (representing the delay for each antenna). This is applicable because as the distance between the source and the antenna array increases, the spherical waves emanating from the source gradually become plane waves. Consequently, the phase difference between adjacent antennas remains constant, resulting in a delay that is solely determined by the angle of incidence.

Conversely, in the near-field regions, the assumption of plane waves is not applicable. As such, it cannot be presumed that the phase difference between adjacent antennas remains constant, making the angle insufficient to compute the delay. Instead, it is imperative to explicitly determine the position of the source to calculate the delay.

Thus, we introduce a novel modification to the steering vector, which now relies on the (x,y) coordinates instead of the θ angle. Specifically, for a source situated at the coordinates (x_1,y_1) the element in the steering vector related to the antenna located at the coordinates (x_2,y_2) can be expressed as follows:

$$e^{-j2\pi\frac{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}{\lambda}} \tag{14}$$

Then for an antenna array where each antenna is located at (x_i,y_i), the steering vector of a source located at (x,y) is:

$$\left[e^{-j2\pi\frac{\sqrt{(x-x_1)^2+(y-y_1)^2}}{\lambda}},\dots,e^{-j2\pi\frac{\sqrt{(x-x_N)^2+(y-y_N)^2}}{\lambda}}\right] \tag{15}$$

VI. Modified Beamforming

Given sample x(t), and steering vector w(x,y), produce y(t):

$$y(t)=w(x,y)^T x(t) \tag{16}$$

Given K samples of the antennas' readings $\{x(t_k)\}_{k=1}^{K}$, try to maximize the average output power which is:

$$P(x,y)=w(x,y)^T\widehat{R_{xx}}w(x,y) \tag{17}$$

Where R_{xx} is the revaluation of the covariance matrix.

Define the steering vector for each (x,y) coordinate in the scanned space:

$$w(x,y)=\left[e^{-j2\pi\frac{\sqrt{(x-x_1)^2+(y-y_1)^2}}{\lambda}},\dots,e^{-j2\pi\frac{\sqrt{(x-x_N)^2+(y-y_N)^2}}{\lambda}}\right] \tag{18}$$

The algorithm computes P(x,y) and identifies the coordinates with the highest values. It then associates each coordinate with its corresponding θ value.

A. Performance

The performance of the Beamforming algorithm is assessed in the context of a single source operating at a frequency of 30 GHz (1 cm wavelength). The source is positioned at an angle of *θ=30°* relative to an antenna array consisting of five elements, each with a separation distance of 0.5 cm (half a wavelength). The antenna array exhibits a signal-to-noise ratio (SNR) of 20[dB]. The near-field to far-field transition distance is established as 8 cm based on the given parameters.

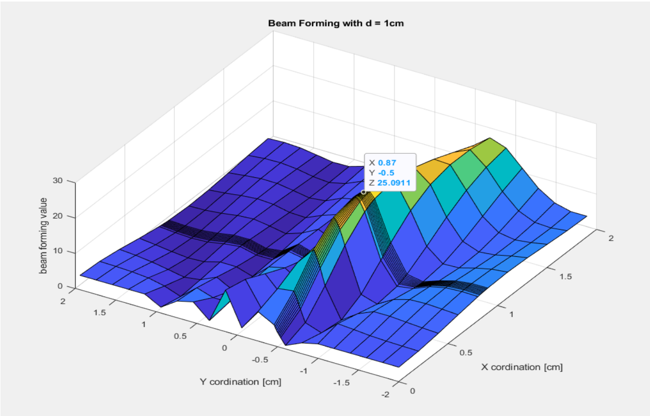

Fig. 4. Beamforming results at a distance of 1cm.

The results demonstrate that the peak point is at (0.87,-0.5), and the angle is:

$$\arctan\left(\frac{-(-0.5)}{0.87}\right)=29.88^\circ$$

The error is 0.12 degrees, and the peak value is close to 25.

VII. Modified Music

Given M sources, N elements, and K samples with value X, the revaluation of the covariance matrix is:

$$\widehat{R_{xx}}=\frac{1}{K}XX^H \tag{19}$$

Sorting the N eigenvalues from the smallest to the largest, so the noise subspace E_N is composed of the N-M eigenvectors associated with the noise:

$$E_N=\left[v_M,v_{M+1},\dots,v_N\right] \tag{20}$$

Then scans angles $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$, and for each scanned angle the steering vector corresponding to this angle is defined as:

$$e(x,y)=\left[e^{-j2\pi\frac{\sqrt{(x-x_1)^2+(y-y_1)^2}}{\lambda}},\dots,e^{-j2\pi\frac{\sqrt{(x-x_N)^2+(y-y_N)^2}}{\lambda}}\right] \tag{21}$$

Subsequently, we compute the product of the steering vector with E_N:

$$\text{MusicSpectrum}(x,y) = \frac{1}{\left| e(x,y)^H E_N E_N^H e(x,y) \right|} \quad (22)$$

Lastly, the algorithm returns the angles associated with the highest values.

A. Performance

The performance of the MUSIC algorithm is assessed in the context of a single source operating at a frequency of 30 GHz (1 cm wavelength). The source is positioned at an angle of θ=30° relative to an antenna array consisting of five elements, each with a separation distance of 0.5 cm (half a wavelength). The antenna array exhibits a signal-to-noise ratio (SNR) of 20[dB]. The near-field to far-field transition distance is established as 8 cm based on the given parameters.

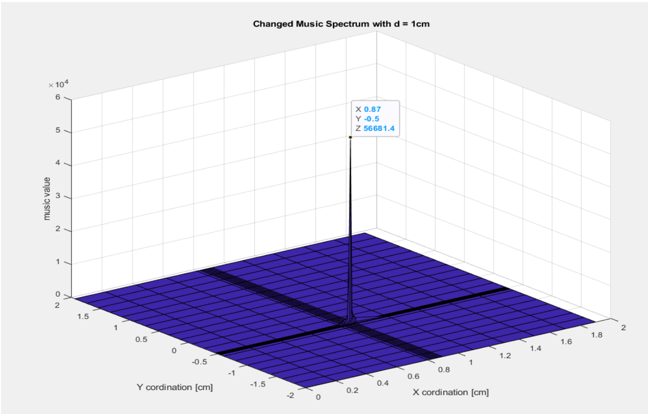

Fig. 5. MUSIC results at a distance of 1cm.

The results demonstrate that the peak point is at (0.87,-0.5), and the angle is:

$$\arctan\left(\frac{-(-0.5)}{0.87}\right) = 29.88^\circ$$

The error is 0.12 degrees, and the peak value is in the order of 10^4.

Conclusion

This paper outlines a modification done to classical beamforming and MUSIC algorithms, which were originally designed for far-field scenarios, and demonstrates their applicability in near-field scenarios. The proposed algorithms not only provide the direction of arrival (DOA), but also explicitly determine the target coordinates. To further improve target identification in the beamforming algorithm, we suggest utilizing the readings provided by the radar to approximate the target's distance and calculate the value in the appropriate coordinates. Furthermore, we point out that our modification technique, which involves swapping the steering vector, can be applied to other super-resolution algorithms that can handle multiple coherent sources, as the MUSIC algorithm is not capable of doing so. It's important to note that the modified algorithms have a higher complexity than the original ones, due to scanning in two dimensions instead of one. To mitigate this limitation, follow-up work can be done to optimize the algorithm by identifying potential target locations, thereby conducting extensive scans exclusively around these points.

Acknowledgment

We would like to express our sincere gratitude to our supervisor, Denis Dikarov, for providing us with valuable guidance and support throughout this research project. He has been instrumental in shaping our research methodology, providing critical feedback on our analysis, and helping us to navigate challenges and roadblocks along the way. His expertise, dedication, and encouragement have been invaluable to our success. We are also grateful to Technion Israel Institute of Technology especially SIPL and the communication lab for providing us with the resources and facilities necessary to carry out this research. Without Denis's guidance and the support of the Technion, this project would not have been possible.

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