

Data-driven Stable State Detection in Power System

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Abstract—Voltage stability is a critical concern in modern power systems as renewable integration, variable loads, and increasingly complex grid topologies require fast and accurate assessment. Classical model-based tools such as continuation power flow are computationally intensive and therefore ill-suited for real-time decision-making. This paper proposes a data-driven voltage-stability framework for small- to medium-scale networks based on Dynamic Mode Decomposition (DMD). We generate pseudo-dynamic snapshot sequences by applying sequential stochastic load perturbations across buses, turning quasi-steady-state power-flow solutions into informative trajectories that capture how demand propagates through the network. DMD is employed to construct a feature representation of system behavior, which is then used to train a one-class voltage stability detector, enabling low-overhead online inference. Numerical results on standard IEEE benchmarks show that combining DMD with PCA yields compact features and markedly improves the separation between stable and unstable operating points, achieving high classification accuracy at substantially reduced computational cost compared to classical scanning methods.

I. INTRODUCTION

The modern power grid has evolved from centralized synchronous generation to decentralized networks dominated by inverter-based resources, resulting in reduced system inertia, increased dynamic loading, and bidirectional power flows, all of which amplify the grid's sensitivity to load variations and disturbances. Consequently, voltage stability has become a central concern for ensuring reliable grid operation [1], as highlighted by recent major blackout events [2]–[5], where inadequate stability assessment has led to severe economic and societal impacts.

In operational practice, voltage-stability assessments are typically conducted by determining safe voltage operation limits. System operators use SCADA (Supervisory Control and Data Acquisition) data measurements and perform offline analysis by solving continuation power-flow (CPF) [6], [7]. In this process, CPF evaluates system behavior by incrementally increasing load along a predefined direction and repeatedly solving the nonlinear power-flow equations until a stability limit is reached. This approach inherently characterizes voltage stability at discrete operating points which are then integrated into predefined remedial-action schemes [8], [9]. Another common analysis is based on acquired data from Phasor Measurement Units (PMUs). Recent methods employed by utilities and research frameworks include Thevenin-equivalent tracking [10], neural-network-based stability assessment, and critical-bus detection [11], all aimed at enabling real-time stability monitoring.

However, the point-wise CPF evaluation offers limited insight into how voltage behavior evolves across a range of operating points, especially under uncertain conditions, and is impractical for real-time operations. CPF repeatedly solves nonlinear power-flow equations over incremental load steps, requiring full Jacobian factorization at each iteration. As shown in [12], the resulting computational burden scales as $\mathcal{O}(MN_B^3)$, where M is the number of load increments and N_B the number of buses. Although recent work has sought to reduce this burden [13], convergence remains sub-optimal for medium-scale grids. Furthermore, although recent PMU-based approaches have demonstrated promise, their high financial cost and reliance on synchronized measurements, high-speed communication, and continuous data management limits widespread deployment [14].

To address these challenges, this work introduces a data-driven voltage-stability framework based on Dynamic Mode Decomposition (DMD), a method originally developed in the fluid-mechanics community, widely used to extract coherent spatial-temporal patterns from system dynamics, by identifying the linear operator that best advances one snapshot to the next (refer to [15], [16] for a comprehensive review of the theoretical foundations and practical implementations of DMD). Beyond its theoretical foundations, DMD has long been employed in settings where first principles models are elusive from turbulent flows to large-scale power grids as shown in [17]–[20]. DMD effectively linearises nonlinear behaviour, enabling eigen decomposition even in very high-dimensional state spaces. In our work, the DMD is directly applied to readily available voltage RMS measurements obtained from different sensors, such as standard PMUs, digital fault recorders, SCADA or Remote Terminal Units (RTUs) systems, followed by dimensionality reduction via Principal Component Analysis (PCA). The framework is fed with time-series of voltage operational points, enabling accurate stability detection, as well as broader insight into system stability compared to recently practiced methods. The system's stability is determined through a fast and feasible one-class classifier inference.

Our voltage stability analysis results demonstrate high accuracy on the IEEE 30-, 118- and 300-bus benchmarks. For example, on the IEEE 118-bus benchmark, our proposed DMD - PCA framework achieves a 95% true-positive rate with only a 5% false-positive rate, while PCA alone requires a 65% false-positive rate to reach the same detection level. These results indicate that DMD captures dynamic information beyond static variance, enabling reliable separation between

stable and instability-prone regimes.

II. CONTINUATION POWER-FLOW STABILITY DETECTION

The real and reactive power injections at bus i are given by [21, Sec. 6.4]

$$P_i = \sum_{j=1}^{N_B} V_i V_j \left[G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \right], \quad (1)$$

$$Q_i = \sum_{j=1}^{N_B} V_i V_j \left[G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j) \right], \quad (2)$$

where N_B is the number of buses in the network, V_i and V_j denote bus-voltage magnitudes; δ_i and δ_j are their phase angles; and G_{ij} , B_{ij} are the real and imaginary elements of the bus-admittance matrix.

In practice, the voltage magnitudes V_i and phase angles δ_i are measured or estimated from SCADA or PMU data, while the active and reactive power injections P_i and Q_i are calculated from these measurements by solving the nonlinear set $\{P_i, Q_i\}_{i=1}^{N_B}$ in (1)-(2).

Usually the iterative Gauss-Newton algorithm for solving the full AC power-flow equations is used [22]. Starting from an initial guess $\mathbf{x}^{(0)} = [\delta^{(0)}, V^{(0)}]^\top$, each iteration linearises the mismatch vector $\mathbf{F}(\mathbf{x}) = [P_i(\mathbf{x}) - P_i^*, Q_i(\mathbf{x}) - Q_i^*]_{i=1}^{N_B}$ by the usage of the Jacobian $\mathbf{J}$ and solves for the correction $\Delta\mathbf{x}$:

$$\underbrace{\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}}_{-\mathbf{F}} = \underbrace{\begin{bmatrix} J_{p\delta} & J_{pV} \\ J_{q\delta} & J_{qV} \end{bmatrix}}_{\mathbf{J}} \underbrace{\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix}}_{\Delta\mathbf{x}}, \quad (3)$$

where ΔP and ΔQ are the changes in the real and reactive powers respectively, and $\Delta\delta$ and ΔV are the deviations in bus voltage angles and bus voltage magnitudes respectively. The Jacobian $\mathbf{J}$ comprises four sub-matrices, $\mathbf{J}_{p\delta}$, $\mathbf{J}_{pV}$, $\mathbf{J}_{q\delta}$, $\mathbf{J}_{qV}$, which quantify the sensitivities of real and reactive power to small perturbations in angles and magnitudes, respectively. Once the correction term $\Delta\mathbf{x}$ is determined, then the new state $\mathbf{x}$ is updated as $\mathbf{x} \leftarrow \mathbf{x} + \Delta\mathbf{x}$, and the process continues by computing the new Jacobian at the new state point, until a predefined stopping criterion is achieved.

Stability is related to the singularity of the Jacobian. As its singularity or near-singularity indicates the loss of solvability of the power-flow equations, marking the onset of voltage collapse. However, each step of the iterative process requires a matrix factorization that involves $\mathcal{O}(N_B^3)$ computations.

Such a power-flow (PF) solution corresponds to *one single operating point* only. It does not directly reveal the behavior of the surrounding region in state space, so exploring the entire stability margin requires repeatedly solving PF equations for many perturbed load conditions. *Continuation power-flow* addresses this by scanning operating points step by step, but this multiplies the computational burden.

Still, computing a full trajectory of M incremental load steps costs $\mathcal{O}(M N_B^3)$. Such iterative solutions are highly expensive and may be impractical for real-time monitoring of large grids.

III. MARKOVIAN-BASED STATE MODEL

We represent the power network by the weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, N_B\}$ is the bus set and $\mathcal{E}$ the branch set [23]. Our goal is to sample the neighborhood of a nominally stable operating point $(\mathbf{V}^*, \mathbf{P}^*, \mathbf{Q}^*)$ in the (V, P, Q) -state space by applying small, stochastic load perturbations that evolve as a Markov process on $\mathcal{G}$.

At iteration $t \in \{1, \dots, k\}$ every perturbation “stick” holds a single bus index $b_t \in \mathcal{V}$. Bus selection follows a first-order, time-homogeneous Markov chain with $\Pr(b_{t+1} = j \mid b_t = i) = 1/\deg(i)$ given that there is a branch between the two nodes; the next perturbed bus depends only on the current bus.

At the start of each Monte-Carlo scenario S , we draw *two* fixed perturbation magnitudes $\Delta P^{(s)} \sim \mathcal{U}[P_{\min}, P_{\max}]$ and $\Delta Q^{(s)} \sim \mathcal{U}[Q_{\min}, Q_{\max}]$, and keep them constant for all k iterations: $\Delta P_t^{(s)} = \Delta P^{(s)}$ and $\Delta Q_t^{(s)} = \Delta Q^{(s)}$.

We launch l active-power sticks *and*, independently, l reactive-power sticks, $2l$ Markov chains in total. The two types are statistically independent and share no information beyond the common graph. Let $\mathbf{b}_t^P = [b_t^{P,1}, \dots, b_t^{P,l}]^\top$ and $\mathbf{b}_t^Q = [b_t^{Q,1}, \dots, b_t^{Q,l}]^\top$ be the active and reactive indices at iteration t . Each component evolves as $b_{t+1}^{P,r} \sim \mathcal{N}(b_t^{P,r})$ and $b_{t+1}^{Q,r} \sim \mathcal{N}(b_t^{Q,r})$, where $\mathcal{N}(i) = \{j \mid (i, j) \in \mathcal{E}\}$. The corresponding updates are:

$$P_{b_t^{P,r}} \leftarrow P_{b_t^{P,r}} + \Delta P^{(s)}, \quad Q_{b_t^{Q,r}} \leftarrow Q_{b_t^{Q,r}} + \Delta Q^{(s)}, \quad r = 1, \dots, l.$$

This neighbor-to-neighbor hand-off lets the disturbance migrate spatially across the grid.

Because sticks evolve independently, two active (or two reactive) sticks could target the same bus at the same iteration. For a bus j the stationary hit-probability is $\pi_j = \frac{\deg(j)}{2|\mathcal{E}|}$. With l sticks of one type, $X_j \sim \text{Binomial}(l, \pi_j)$ counts simultaneous hits. The per-bus collision probability is

$$\Pr\{\text{collision at } j\} = 1 - (1 - \pi_j)^l - l\pi_j(1 - \pi_j)^{l-1}. \quad (4)$$

For each benchmark we substitute the stationary hit-probability of the highest-degree bus into (4); we also estimate the *expected number of collisions per iteration*

$$\mathbb{E}[\#\text{collisions}] = \frac{l(l-1)}{2} \sum_{j=1}^{N_B} \pi_j^2.$$

The collision probability per bus remains below 6.2% for the 30-bus grid and below 1% for the 118- and 300-bus systems. Because a collision merely applies a second equal increment to the same bus, such rare events have a negligible impact and are ignored in subsequent analysis.

IV. PROPOSED STABLE STATE DETECTOR

Unlike its conventional use for modal decomposition or forecasting, here DMD is repurposed as a *stability detector*: a novel application that leverages its spectral sensitivity to identify deviations from nominal operating regimes.

In the present work, the role of clock time is played by a sequence of incremental active and reactive-power perturbations, so the resulting modes describe how voltage profiles

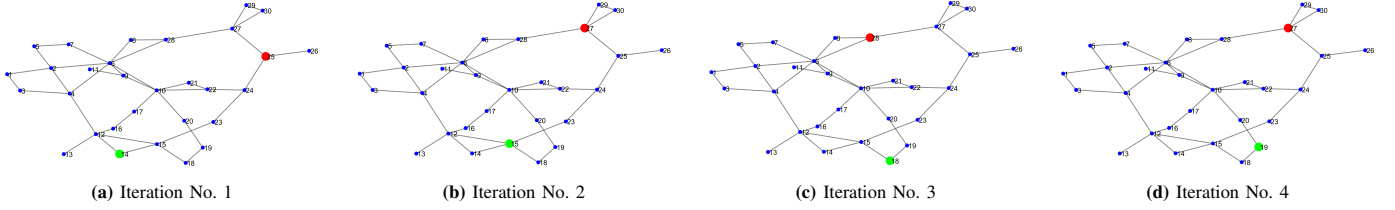


Fig. 1: Illustration on the IEEE 30-bus grid: four consecutive load-perturbation iterations. In each panel, blue buses remain unchanged, red buses receive an active-power increment ΔP , and green buses receive a reactive-power increment ΔQ , revealing the spatial propagation of the disturbance.

transform as loading conditions vary in the system. Fig. 1 traces four consecutive iterations for $l = 2$. At iteration 1, an active-power stick perturbs bus 25 and a reactive-power stick perturbs bus 14. At iteration 2 each stick hands its “load stick” to one random neighbor: the active increment moves to bus 27, and the reactive increment to bus 15. The process continues for the remaining $k - 2$ steps, emulating realistic, spatially propagating load variations.

Each Monte-Carlo scenario j generates a voltage-response matrix $\mathbf{S}_j$,

$$\mathbf{S}_j = [\mathbf{v}^{(1)} \quad \mathbf{v}^{(2)} \quad \dots \quad \mathbf{v}^{(k)}]_{N_B \times k} \quad (5)$$

where $\mathbf{v}_j^{(i)} \in \mathbb{R}^{N_B}$, $i = 1, \dots, k$ stores the amplitude of the voltage of all the buses in the power grid at the i -th iteration. The matrix $\mathbf{S}_j$ is referred as “snapshot” with label $y_j \in \{0, 1\}$, where ‘1’ denotes a stable condition.

Because real grids operate close to equilibrium, we assume that there exists a latent space, which we seek to exploit, in which stable snapshots concentrate into a tight, near-zero residual cluster. Any new snapshot that falls far outside this cluster is flagged as a potential voltage-instability state.

The workflow is divided into two phases: a *training* basis-construction and threshold-selection stage, where snapshots $\{\mathbf{S}_j\}$ are used to learn the linear mapping that projects the data onto a representation subspace using the DMD and then to a low-dimensional reference subspace using the PCA, and an *online* classification stage, where new snapshots are evaluated as *test data* against the threshold derived during the *training* phase.

A. Training Phase

For each snapshot $\mathbf{S}_j$, $j = 1, \dots, m_{\text{train}}$, composed of voltage-responses measurements collected over k steps (5), the matrix $\Phi(\mathbf{S}_j) \in \mathbb{C}^{N_B \times r}$ is obtained by applying DMD, which decomposes the voltage trajectories into a set of spatial modes that capture the dynamic patterns governing the system response. In this work, only the r most significant modes are retained, corresponding to a low-rank DMD representation, as in previous work [19]. This matrix is then vectorized and its element-wise magnitude is taken to form the feature vector $\tilde{\mathbf{v}}(\mathbf{S}_j) = |\text{vec}(\Phi(\mathbf{S}_j))| \in \mathbb{R}^{N_B r}$ where $|\cdot|$ denotes element-wise absolute value. Then the feature vectors extracted from all training snapshots are stacked column-wise to construct the training matrix

$$\mathbf{P} = [\tilde{\mathbf{v}}^{(1)}, \tilde{\mathbf{v}}^{(2)}, \dots, \tilde{\mathbf{v}}^{(m_{\text{train}})}] \in \mathbb{R}^{N_B r \times m_{\text{train}}}.$$

To reduce the memory required to store the projection operator and to suppress noise, PCA is applied to $\mathbf{P}$ by retaining a rank- d orthonormal basis yielding the projection matrix $\mathbf{U} \in \mathbb{R}^{N_B r \times d}$, $\mathbf{U}^\top \mathbf{U} = \mathbf{I}_d$, which captures the dominant variance associated with stable operating conditions. A validation-driven threshold for the one-class classifier used during inference is selected using a validation dataset comprising both stable and unstable snapshots.

For each snapshot $\mathbf{S}_i'$, $i = 1, \dots, m_{\text{validation}}$ the corresponding feature vector $\tilde{\mathbf{v}}^{(s')}$ is constructed as described above, and its reconstruction error under the learned projection is computed as $e = \|\tilde{\mathbf{v}}^{(s')} - \mathbf{U} \mathbf{U}^\top \tilde{\mathbf{v}}^{(s')}\|_2$.

Let $\mathcal{E}^+$ and $\mathcal{E}^-$ be the multi sets of reconstruction errors for stable and unstable validation snapshots, respectively. Choose the operating threshold T^* to minimize the validation misclassifications rate, i.e.,

$$T^* = \arg \min_{T > 0} \underbrace{\frac{1}{|\mathcal{E}^+|} \sum_{e \in \mathcal{E}^+} \mathbf{1}_{e > T}}_{\text{FN}(T)} + \underbrace{\frac{1}{|\mathcal{E}^-|} \sum_{e \in \mathcal{E}^-} \mathbf{1}_{e < T}}_{\text{FP}(T)}. \quad (6)$$

Other choices of T^* that maximizes validation accuracy are also possible.

B. Inference Phase

Given a *new* test snapshot $\mathbf{S}''$, and the projection matrix $\mathbf{U}$ obtained during the training phase, the corresponding feature vector $\tilde{\mathbf{v}}^{(s'')}$ is constructed following the same procedure described in the training phase, and its reconstruction error under the learned projection is computed as

$$e = \|\tilde{\mathbf{v}}^{(s'')} - \mathbf{U} \mathbf{U}^\top \tilde{\mathbf{v}}^{(s'')}\|_2$$

The operating condition is then classified by applying the fixed decision threshold

$$\text{classify}(\mathbf{S}'') = \begin{cases} \text{stable}, & e \leq T^*, \\ \text{unstable}, & e > T^*. \end{cases}$$

C. Inference Computation Load

Let N_B be the number of buses, k the length of the load-perturbation trajectory used to build one test snapshot ($\mathbf{S} \in \mathbb{C}^{N_B \times k}$), $r \ll k$ the number of DMD modes, and $d \ll N_B r$ the number of principal components in the basis $\mathbf{U}$.

The online processing of a single snapshot consists of several computational steps. First, in the DMD stage, the truncated singular-value decomposition of the snapshot matrix $\mathbf{S}$

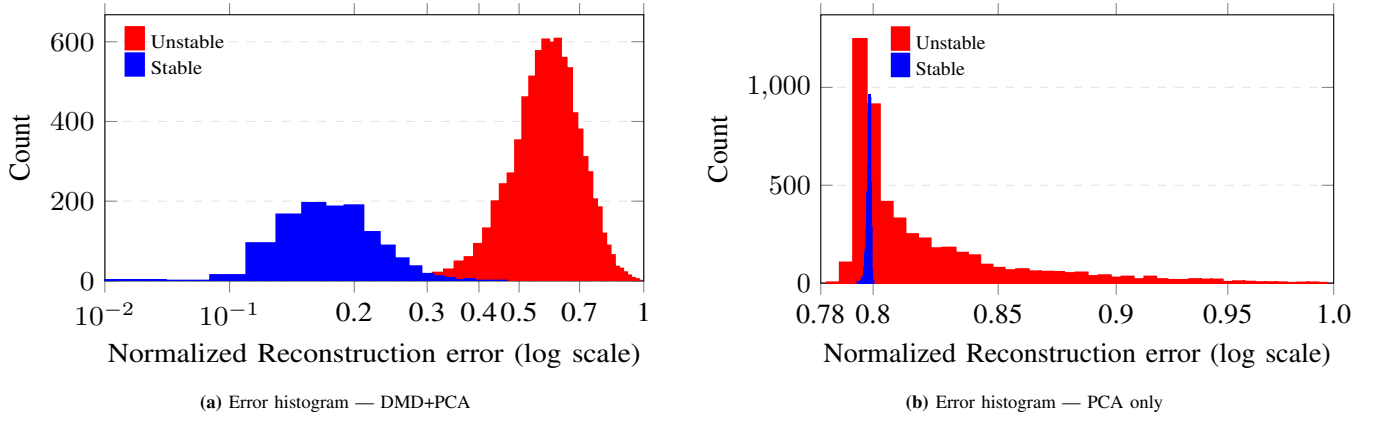


Fig. 2: Performance of the proposed stability classifier. Normalised reconstruction-error distributions with and without DMD.

incurs a cost of $\mathcal{O}(N_B^2 k)$, since in practice $N_B \simeq k$. Forming the reduced operator and computing its eigen-decomposition adds an additional $\mathcal{O}(r^3)$ [24]. Next, vectorization of the DMD mode matrix $\Phi \in \mathbb{C}^{N_B \times r}$ requires $\mathcal{O}(N_B r)$ operations. The subsequent projection and reconstruction step, in which $\hat{\mathbf{v}} = \mathbf{U}\mathbf{U}^T \mathbf{v}$ is computed, involves two dense matrix-vector multiplications and thus scales as $\mathcal{O}(N_B r d)$. Finally, evaluating the reconstruction residual norm adds a further $\mathcal{O}(N_B r)$ computational cost.

Consequently, processing a single snapshot entails $\mathcal{O}(N_B^2 k + r^3 + N_B r + N_B r d)$. In practice, this workload is governed by the first term, the truncated SVD of the $N_B \times k$ snapshot matrix, because the secondary ranks satisfy $r, d \ll N_B \lesssim k$. However, the overall effort scales only quadratically in N_B and linearly in k , an improvement over the $\mathcal{O}(N_B^3 k)$ operations demanded by a full continuation of the power-flow scan at every operating point.

Storing the projection basis $\mathbf{U}$ for the online stage requires only $\mathcal{O}(N_B r d)$ memory, a budget that is independent of the size of the training set.

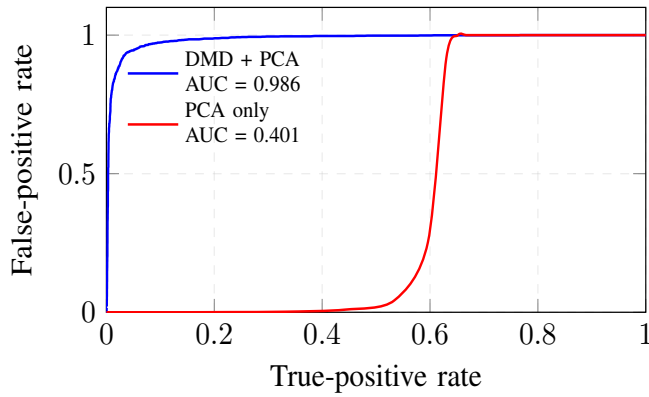
V. NUMERICAL RESULTS

Voltage snapshots were generated with *MATPOWER* [25] for the IEEE 30-, 118-, and 300-bus systems using $k = 90, 200$, and 500 iterations, respectively. For each snapshot, the stick count is drawn from a system-specific uniform law: $l_{30} \sim \mathcal{U}[1, 5]$, $l_{118} \sim \mathcal{U}[2, 7]$, and $l_{300} \sim \mathcal{U}[3, 10]$. A snapshot $\mathbf{S}$ is labeled *stable* when the Newton-Raphson power flow converges for all k perturbation steps. All other snapshots are labeled **unstable**. The labeled data set is divided into three disjoint subsets. *Stable* snapshots are used to construct the DMD - PCA reference subspace during training. A separate validation set, containing both *stable* and *unstable* snapshots, is used only to select the decision threshold T^* in (6). An independent test set, containing *stable* and *unstable* snapshots that were not used for either basis construction or threshold selection. The plots in Fig. 2 and 3 correspond to the IEEE 118-bus benchmark; nearly identical patterns were observed on the 30-bus and 300-bus grids, confirming that the findings are generalizable across system sizes. The

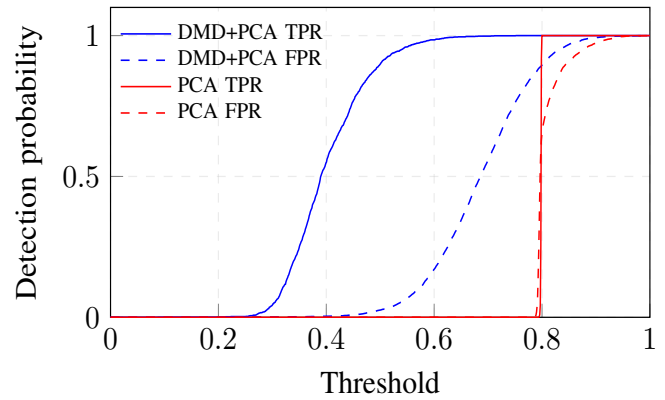
figures present four complementary performance views: the normalized reconstruction-error histograms (Fig. 2a-2b), the *receiver operating characteristic* (ROC) curve (Fig. 3a), and the threshold sensitivity sweep (Fig. 3b).

- **Normalized error distributions** Sub-fig. 2a shows that, after the dynamic mode-extraction step, stable snapshots cluster tightly closer to zero error, whereas unstable snapshots form a distinct lobe closer to 1. By contrast, Sub-fig 2b demonstrates that a PCA-only baseline fails to discriminate: the two classes overlap almost completely, underscoring that static variance alone is insufficient and that the discriminative information captured by DMD is inherently dynamic.
- **ROC performance** The ROC curve in Fig. 3a traces the trade-off between true-positive rate (TPR) and false-positive rate (FPR) as the decision threshold sweeps the $[0, 1]$ error range. The DMD-PCA classifier achieves an area under the curve (AUC) of 0.986, approaching perfect separability, whereas PCA alone reaches only $\text{AUC} = 0.401$. This ROC profile for DMD-PCA indicates that the modes extracted from sequential $\Delta P, \Delta Q$ perturbations encode decisive precursors of voltage collapse.
- **Threshold-sensitivity sweep** Fig. 3b shows how the fraction of test snapshots labelled *stable* changes with the decision threshold T . With the DMD - PCA classifier the true-positive rate reaches 95% when $T = 0.5395$, at which point the corresponding false-positive rate is still only 5%, giving operators a wide and reliable operating margin. By contrast, the PCA-only curve does not reach a 95% true-positive rate until $T = 0.7998$, where the false-positive rate has already risen to 65%. This stark difference underscores the added discriminative power conferred by the dynamic modes extracted with DMD.

As shown in Fig. 2 and 3, the proposed DMD-PCA workflow delivers a markedly cleaner separation between stable and unstable operating points than a dimensionality-reduction scheme based on PCA alone.



(a) ROC curve (TPR vs. FPR), both methods



(b) Stable-label rate vs. threshold, both methods

Fig. 3: Left: ROC curve. Right: fraction of test snapshots classified as stable as the decision threshold varies.

VI. CONCLUSION

The proposed DMD - PCA approach delivers a low-overhead screening tool for voltage-stability assessment on small- and medium - sized networks. This represents a novel use of DMD not merely as a dynamic decomposer but as a data-driven *detector* of stability margins, exploiting its eigenstructure to discriminate between secure and collapse-prone regimes. By operating on blocks of voltage snapshots rather than on a single operating point, the proposed DMD - PCA framework achieves improved detection performance while maintaining favorable computational efficiency. The online computation is limited to a single truncated SVD and a small number of vector operations, enabling near real-time situational awareness for power grids with up to a few hundred buses.

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